XIX. *The Residual Charge of the Leyden Jar.* *By* J. HOPKINSON, *M.A., D.Sc. Communicated by Sir* WILLIAM THOMSON, *F.R.S., Professor of Natural Philosophy in the University of Glasgow.*

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1. SUPPOSE that the state of a dielectric under electric force* is somewhat analogous to that of a magnet, that each small portion of its substance is in an electropolar state. Whatever be the ultimate physical nature of this polarity, whether it arises from conduction, the dielectric being supposed heterogeneous (see MAXWELL'S 'Electricity and Magnetism,' vol. i. arts. 328–330), or from a permanent polarity of the molecules analogous to that assumed in WEBER'S theory of induced magnetism, the potential at points external to the substance due to this electropolar state will be exactly the same as that due to a surface distribution of electricity, and its effect at all external points may be masked by a contrary surface distribution. Assume, further, that dielectrics have a property analogous to coercive force in magnetism, that the polar state does not instantly attain its full value under electric force, but requires time for development and also for complete disappearance when the force ceases. The residual charge may be explained by that part of the polarization into which time sensibly enters. A condenser is charged for a time, the dielectric gradually becomes polarized; on discharge the two surfaces of the condenser can only take the same potential if a portion of the charge remain sufficient to cancel the potential, at each surface, of the polarization of the dielectric. The condenser is insulated, the force through the dielectric is insufficient to permanently sustain the polarization, which therefore slowly decays; the potentials of the polarized dielectric and of the surface charge of electricity are no longer equal, the difference is the measurable potential of the residual or return charge at the time. It is only necessary to assume a relation between the electric force, the polarization measured by the equivalent surface distribution, and the time. For small charges a possible law may be the following:—For any intensity of force there is a value of the polarization proportional to the force to which the actual polarization approaches at a rate proportional to its difference therefrom. Or we might simply assume that the difference of potentials E of the two surfaces and the polarization are connected with the time by two linear differential equations of the first order. If this be so, E can be expressed in terms of the time t during insulation by the formula $E=(A+B\varepsilon^{-\mu t})\varepsilon^{-\lambda t}$, where λ and μ

* To define the electric force within the dielectric it is necessary to suppose a small hollow space excavated about the point considered; the force will depend on the form of this space; but it is not necessary for the present purpose to decide what form it is most appropriate to assume.

are constants for the material, and A and B are constants dependent on the state of the dielectric previous to insulation. It should be remarked that λ does not depend alone on the conductivity and specific inductive capacity, as ordinarily determined, of the material, but also on the constants connecting polarization with electric force. Indeed if the above view really represent the facts, the conductivity of a dielectric determined from the steady flow of electricity through it measured by the galvanometer will differ from that determined by the rate of loss of charge of the condenser when insulated.

2. A Florence flask nearly 4 inches in diameter was carefully cleansed, filled with strong sulphuric acid, and immersed in water to the shoulder. Platinum wires were dipped in the two fluids, and were also connected with the two principal electrodes of the quadrant electrometer. The jar was slightly charged and insulated, and the potentials read off from time to time. It was found (1) that even after twenty-four hours the percentage of loss per hour continued to decrease, (2) that the potential could not be expressed as a function of the time by two exponential terms. But the latter fact was more clearly shown by the rate of development of the residual charge after different periods of discharge, which put it beyond doubt that if the potential is properly expressed by a series of exponential terms at all, several such terms will be required.

The following roughly illustrates how such terms could arise. Glass may be regarded as a mixture of a variety of different silicates; each of these may behave differently under electric force, some rapidly approaching the limiting polarity corresponding to the force, others more slowly. If these polarities be assumed to be n in number, they and E may be connected with the time by $n+1$ linear differential equations. Hence uring insulation E would be expressed in the form $\Sigma_0^n A_r \varepsilon^{-\lambda_r t}$. Suppose now a condenser be charged positively for a long time, the polarization of all the substances will be fully developed; let the charge be next negative for a shorter time, the rapidly changing polarities will change their sign, but the time is insufficient to reverse those which are more sluggish. Let the condenser be then discharged and insulated, the rapid polarizations will decay, first liberating a negative charge; but after a time the effect of the slow terms will make itself felt and the residual charge becomes positive, rises to a maximum, and then decays by conduction. This inference from these hypotheses and the form of the curve connecting E with t for a simple case of return charge is verified in the following experiments.

3. A flask was immersed* in and filled with acid to the shoulder. Platinum electrodes communicated with the electrometer as before. The flask was strongly charged positive at 5.30 and kept charged till 6.30, then discharged till 7.8 and negatively charged till 7.15, when it was discharged and insulated. The potential was read off at intervals till 8.20. The abscissæ of curve A (Plate 44) represent the time from insulation, the ordinates the corresponding potentials, positive potentials being measured upwards. It will be seen that a considerable negative charge first appeared, attaining a maximum in about five

* Acid on both sides of the dielectric, that there might be no electromotive force from the action of acid on water either through or over the surface of the glass.

minutes; it then decreased, and the potential was *nil* in half an hour; the main positive return charge then came out, and was still rapidly increasing at 8.20, when the flask was again discharged. At 8.39 the same flask was charged negatively till 8.44, then discharged and charged positively for 45 seconds, insulated 15 seconds and discharged, and finally insulated at 8.45. Curve B (Plate 44) represents the subsequent potentials. It is seen that the return charge twice changes sign before it assumes its final character. The experiment was several times repeated with similar results.

Sir WILLIAM THOMSON has informed the author, since these experiments were tried, that he himself performed similar experiments many years ago, and showed them as lecture illustrations in his Class in the University of Glasgow, but never otherwise published them, proving that the charges come out of the glass in the inverse order to that in which they go in†.

4. When steel is placed in a magnetic field, mechanical agitation accelerates the rapidity with which its magnetic polarity is developed. Again, vibration reduces the magnetism of a magnet, or, so to speak, shakes its magnetism out. This would suggest, on the present hypothesis, that vibration would accelerate changes in the electric polarity of a dielectric, or shake down polarization and liberate residual charge. The following experiments verify this anticipation. The arrangement was as in (3). The flask was strongly charged for some hours, discharged at 4.45 P.M., and kept with the two coatings connected by a platinum wire, except for a few moments at a time, to ascertain the rate at which the polarization was decaying, till 9.48, when the flask was insulated and the number of seconds observed in which the potential rose to 100, 200, 300, and 500 divisions of the scale of the quadrant electrometer, every thing being as steady as possible. The flask was then discharged, again insulated at 10.18, and the development of the charge observed, the neck of the flask being sharply tapped during the whole time. The experiment was repeated quiet at 10.48, with tapping at 11.16. Column I. gives the time of beginning the observation, II., III., IV., and V. the number of seconds in which charges 100, 200, 300, 500 developed respectively. The periods of tapping are marked with an asterisk.

I.	II.	III.	IV.	V.
9.48	118	240	367	624
10.18	80*	140*	185*	320*
10.48	140	285	440	750
11.16	120*	210*	310*	540*

The effect may appear small; but it must be remembered that, the flask containing and being immersed in sulphuric acid to the shoulder, the vibration caused by tapping

† These results are closely analogous to those obtained by BOLTZMANN for torsion (Sitzungsberichte der k. Akad. der Wiss. zu Wien, Bd. lxx. Sitzung 8. Oct. 1874). From his formulæ it follows that if a fibre of glass is twisted for a long time in one direction, for a shorter time in the opposite direction, and is then released, the set of the fibre will for a time follow the last twist, will decrease, and finally take the sign of the first twist.

the neck could be but small, and could scarcely penetrate to the lower part of the flask. The experiment was subsequently repeated with the same flask and with a similar result; but it was further found that the effect of tapping was more marked when the periods during which the flask was strongly charged and discharged were long than when they were short. For example, when the flask was charged half an hour, then discharged five minutes, the effect of tapping was very slight although unmistakable. That portion of the return charge which comes out slowly is more accelerated by vibration than that which comes out fast. A comparison of the rates at 10.18 and 11.16 of the above Table also shows that a flask which has been tapped is less susceptible to the effect of tapping than it was before it was touched. In some cases also it was noticed that if three observations were made, the first quiet, the second tapped, and the third quiet, the third charge came out more rapidly than the first. The last experiment on tapping below illustrates both of these points.

A flask was mounted as before, strongly charged at 12 o'clock, discharged at 3, and remained discharged till 5.15, when it was insulated, and the time which the image took to traverse 200 divisions was noted; after passing that point the flask was again discharged. The first column gives the instant of insulation, the second the time of covering 200 divisions. The observations without mark were made with the flask untouched, in those marked * it was sharply tapped all over with a glass rod dipping in the acid, whilst in those marked † the rod was muffled with a piece of india-rubber tubing.

	Time of insulation.		Time occupied in traversing 200 divisions of the scale.	
	h	m	min.	secs.
	5	15	1	23
	5	18	1	23
	*5	21		40
	5	24	1	17
	†5	27		48
	5	30	1	27
Remained discharged till	5	46½	1	25
	†5	49½		54
	5	51½	1	24
	*5	54	1	3
	5	56	1	24
Remained discharged till	6	39	2	7½
	*6	43	1	54
	6	47	2	8
	†6	51	2	2
	6	55	2	13

	Time of insulation. h	m	Time of traversing 100 divisions. min.	secs.
Remained discharged till	8	51	2	11
	†8	55		57
	8	58	2	12
	*9	2	1	2
	9	5	2	14
	†9	9		54
	9	12	2	17

The same flask was strongly charged at 9.15 in the evening and discharged at 9 the following morning, and remained so till 7.13 in the evening, when the following observations of the time of traversing 100 divisions were made:—

Time of insulation. h	m	Time occupied in traversing 100 divisions of the scale. min.	secs.
7	13	2	43
†7	18	1	35
7	21	2	35
*7	25	1	53
7	28	2	27
†7	32	1	46
7	35	2	26
*7	39	1	49
7	43	2	25

The result here was less than the author expected, considering the long period of discharge and the considerable effect obtained in the previous experiment; this may perhaps be due to change of temperature, or perhaps to a difference in the vigour with which the flask was tapped‡.

5. When a charge is given to an insulated flask, owing to polarization the percentage of loss per minute continuously diminishes towards a limiting value. When the flask is charged, discharged, and insulated, one would expect that after attaining a maximum potential the rate of loss would steadily *increase* towards the same limiting value as in the former case. The following experiment shows that this is not always the case.

A flask of window-glass, much more conductive than the Florence flask, was mounted as in (3) and (4). It was charged, and the charge maintained for three quarters of an hour, then discharged for a quarter of an hour, and insulated. In four minutes the charge attained a maximum value 740. In fifteen minutes the potential was 425, in twenty minutes 316, giving a loss in five minutes of 26 per cent. In thirty minutes it was 186, and in thirty-five minutes 146, a loss of $21\frac{1}{2}$ per cent. The intermediate and

‡ It is recorded by Dr. YOUNG that an electical jar may be discharged either by heating it or by causing it to sound by the friction of the finger.

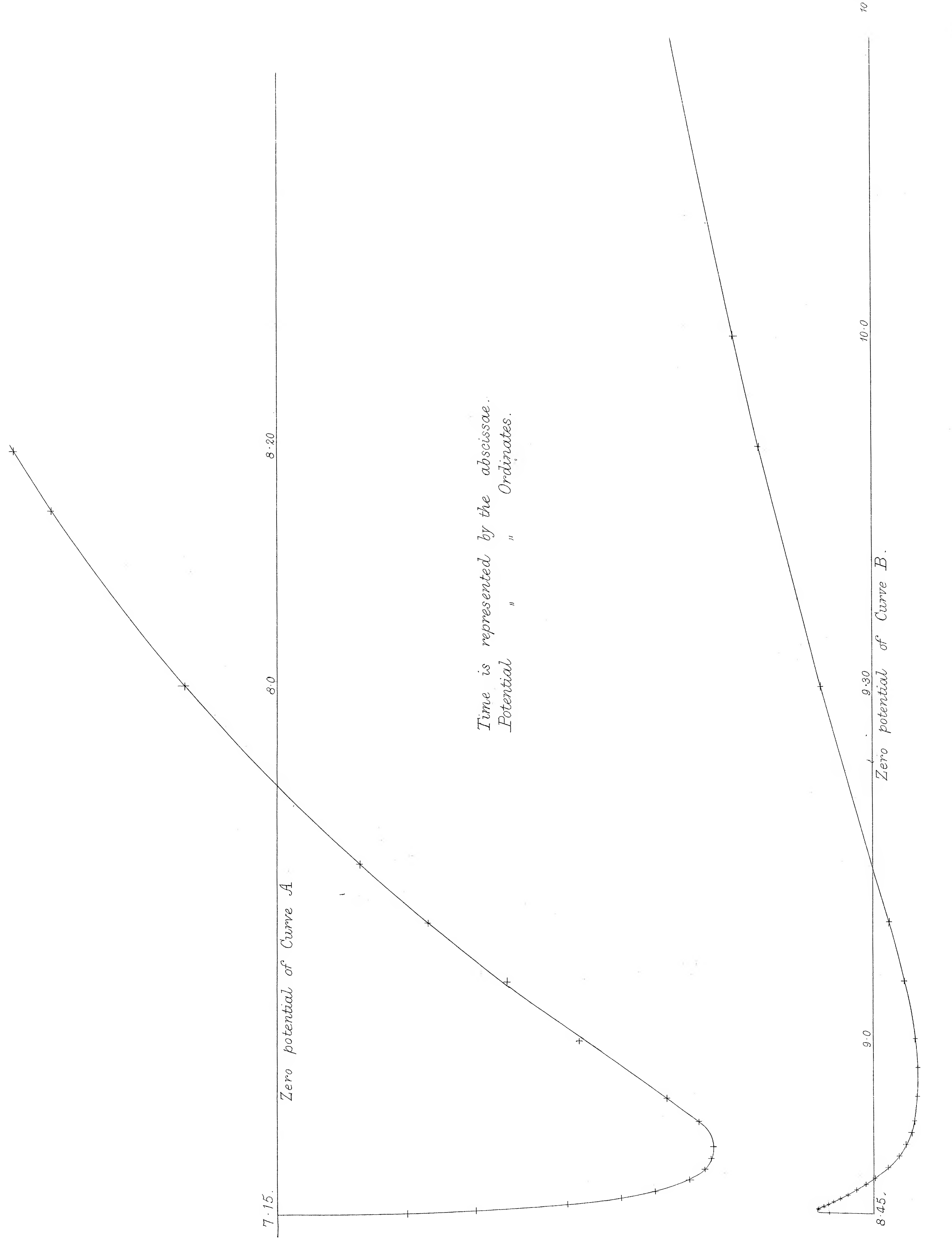
subsequent readings of the same series showed a steady decrease to as little as 15 per cent. The experiment was repeated with the same flask, but with shorter periods of discharge and with a similar result.

6. Although the above view is only proposed as a provisional working hypothesis, some suggestions which it indicates may be pointed out.

Temperature has three effects on the magnetic state of iron or steel:—(1) Changes of temperature cause temporary changes in the intensity of a magnet; (2) temperature affects the "permeability" of a magnet; at a red heat iron is no longer sensibly magnetic; (3) a rise of temperature reduces coercive force.

It may be expected that the polarity of dielectrics may also be affected in three analogous ways:—(1) a sudden change of temperature might directly and suddenly affect the polarity (an example of this we have in the phenomena of pyro-electricity); (2) the constant expressing the ratio of limiting polarity to electromotive force may depend on temperature; and (3) temperature may alter the constant, expressing the rate at which polarity approaches its limiting value for a given force, as it is known to alter the specific conductivity. Mr. PERRY'S experiments show that temperature does affect the polarization of dielectrics, but in which way does not appear.

Sir WILLIAM THOMSON (papers on Electrostatics and Magnetism, art. 43) explains specific inductive capacity by a polarization of the dielectric following the same formal laws as magnetism. It is only necessary to introduce time into that explanation as here proposed to enable it to cover the phenomena of residual charge. Again (see NICHOL'S Cyclopædia), Sir W. THOMSON explains the phenomena of pyro-electricity by supposing that every part of the crystal of tourmaline is electropolar, that temperature changes the intensity of its polarity, and that this polarity is masked by a surface distribution of electricity supplied by conduction over the surface or otherwise. We have, then, in tourmaline an analogue to a rigidly magnetized body, in glass or other dielectrics analogues to iron having more or less coercive force.


Time is represented by the abscissae.
Potential " " Ordinates.
Zero potential of Curve A
Zero potential of Curve B.
7·15.
8·0
8·20
8·45.
9·0
9·30
10·0